Sample Space and Probability

- ***Random experiment:*** its outcome, for some reason, cannot be predicted with certainty.
  - Examples: throwing a die, flipping a coin and drawing a card from a deck.
- ***Sample space:*** the set of all possible outcomes, denoted by S. Outcomes are denoted by E's and each E lies in S, i.e., $E \in S$.
- A sample space can be discrete or continuous.
- Events are subsets of the sample space for which measures of their occurrences, called probabilities, can be defined or determined.

THREE AXIOMS OF PROBABILITY

- For a discrete sample space S, define a probability measure P on as a set function that assigns nonnegative values to all events, denoted by E, in such that the following conditions are satisfied
- Axiom 1: $0 \leq P(E) \leq 1$ for all $E \in S$
- Axiom 2: $P(S) = 1$ (when an experiment is conducted there has to be an outcome).
- Axiom 3: For mutually exclusive events E1, E2, E3,. . . we have

$$P\left(\bigcup_{i=1}^{\infty} E_i\right) = \sum_{i=1}^{\infty} P(E_i).$$

Conditional Probability

- We observe or are told that event E1 has occurred but are actually interested in event E2: Knowledge that of E1 has occurred changes the probability of E2 occurring.
- If it was P(E2) before, it now becomes P(E2|E1), the probability of E2 occurring given that event E1 has occurred.
- This conditional probability is given by

$$P(E_2|E_1) = \begin{cases} \dfrac{P(E_2 \cap E_1)}{P(E_1)}, & \text{if } P(E_1) \neq 0 \\ 0, & \text{otherwise} \end{cases}$$

- If P(E2|E1) = P(E2), or P(E2 ∩ E1) = P(E1)P(E2), then E1 and E2 are said to be statistically independent.
- Bayes' rule
  - P(E2|E1) = P(E1|E2)P(E2)/P(E1)

Mathematical Model for Signals

- Mathematical models for representing signals
  - Deterministic
  - Stochastic
- Deterministic signal: No uncertainty with respect to the signal value at any time.
  - Deterministic signals or waveforms are modeled by explicit mathematical expressions, such as

$$x(t) = 5 \cos(10*t).$$

  - ***Inappropriate for real-world problems???***
- Stochastic/Random signal: Some degree of uncertainty in signal values before it actually occurs.
  - For a random waveform it is not possible to write such an explicit expression.
  - Random waveform/ random process, may exhibit certain regularities that can be described in terms of probabilities and statistical averages.
  - e.g. thermal noise in electronic circuits due to the random movement of electrons

Energy and Power Signals

- The performance of a communication system depends on the received signal energy: higher energy signals are detected more reliably (with fewer errors) than are lower energy signals.
- An electrical signal can be represented as a voltage $v(t)$ or a current $i(t)$ with instantaneous power $p(t)$ across a resistor defined by

$$p(t) = \frac{v^2(t)}{\Re}$$

OR

$$p(t) = i^2(t)\Re$$

Energy and Power Signals

- In communication systems, power is often normalized by assuming R to be 1.
- The normalization convention allows us to express the instantaneous power as

$$p(t) = x^2(t)$$

  where $x(t)$ is either a voltage or a current signal.
- The energy dissipated during the time interval (-T/2, T/2) by a real signal with instantaneous power expressed by Equation (1.4) can then be written as:

$$E_x^T = \int_{-T/2}^{T/2} x^2(t)\, dt$$

- The average power dissipated by the signal during the interval is:

$$P_x^T = \frac{1}{T} E_x^T = \frac{1}{T} \int_{-T/2}^{T/2} x^2(t)\, dt$$

Energy and Power Signals

- We classify $x(t)$ as an *energy signal* if, and only if, it has nonzero but finite energy ($0 < E_x < \infty$) for all time, where

$$E_x = \lim_{T \to \infty} \int_{-T/2}^{T/2} x^2(t)\, dt$$

$$= \int_{-\infty}^{\infty} x^2(t)\, dt$$

- An energy signal has finite energy but *zero average power*
- Signals that are both deterministic and non-periodic are termed as Energy Signals

Energy and Power Signals

- Power is the rate at which the energy is delivered
- We classify $x(t)$ as an *power signal* if, and only if, it has nonzero but finite energy ($0 < P_x < \infty$) for all time, where

$$P_x = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} x^2(t)\, dt$$

- A power signal has finite power but infinite energy
- Signals that are random or periodic termed as Power Signals

Random Variable

- Functions whose domain is a sample space and whose range is a some set of real numbers is called ***random variables.***
- Type of RV's
  - Discrete
    - E.g. outcomes of flipping a coin etc
  - Continuous
    - E.g. amplitude of a noise voltage at a particular instant of time

Random Variables

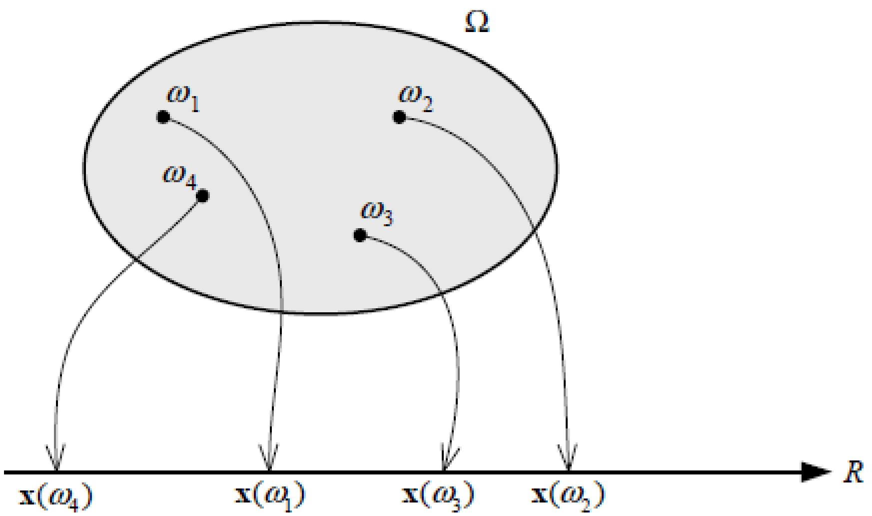
Random Variables

- All useful signals are random, i.e. the receiver does not know a priori what wave form is going to be sent by the transmitter
- Let a *random variable* $X(A)$ represent the functional relationship between a random event A and a real number.
- The *distribution function* $F_x(x)$ of the random variable X is given by

$$F_X(x) = P(X \leq x)$$

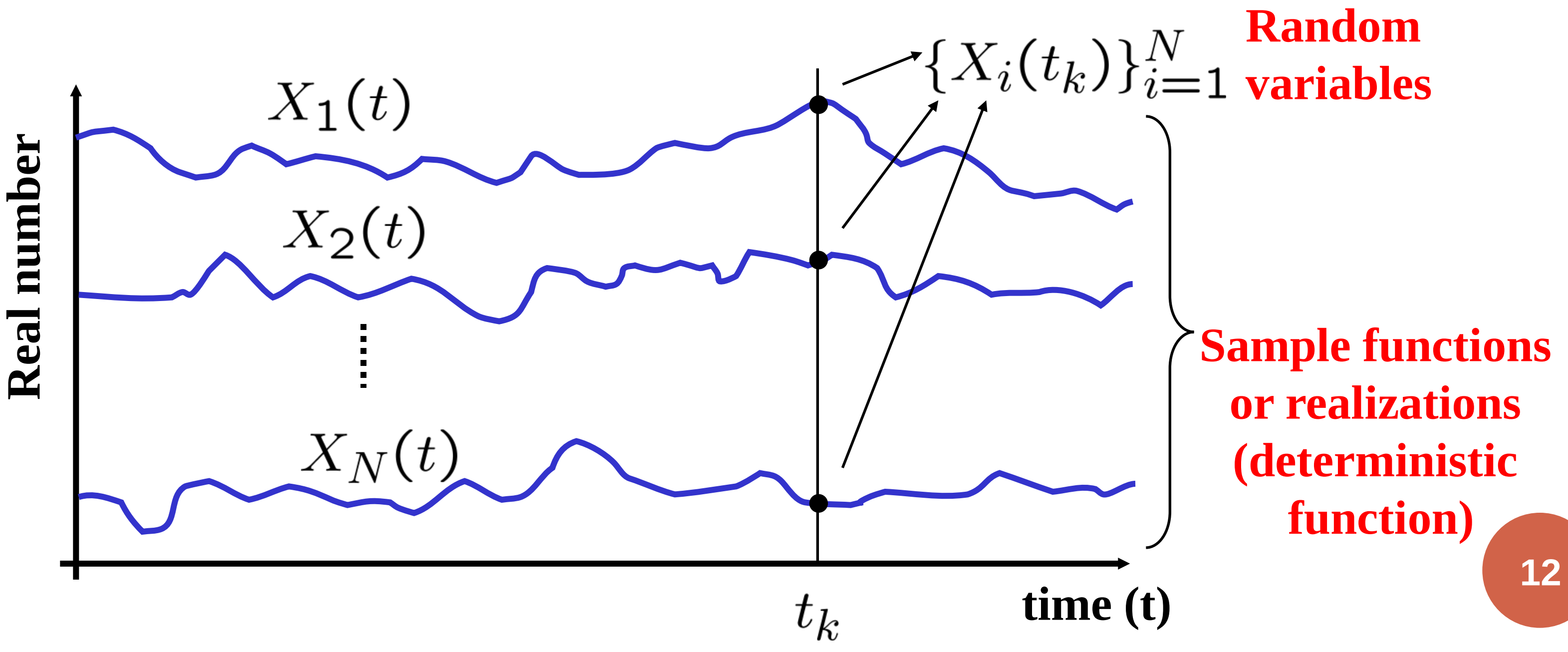
RANDOM VARIABLE

- A random variable is a mapping from the sample space to the set of real numbers.
- We shall denote random variables by boldface, i.e., **x, y,** etc., while individual or specific values of the mapping **x** are denoted by **x**(w).



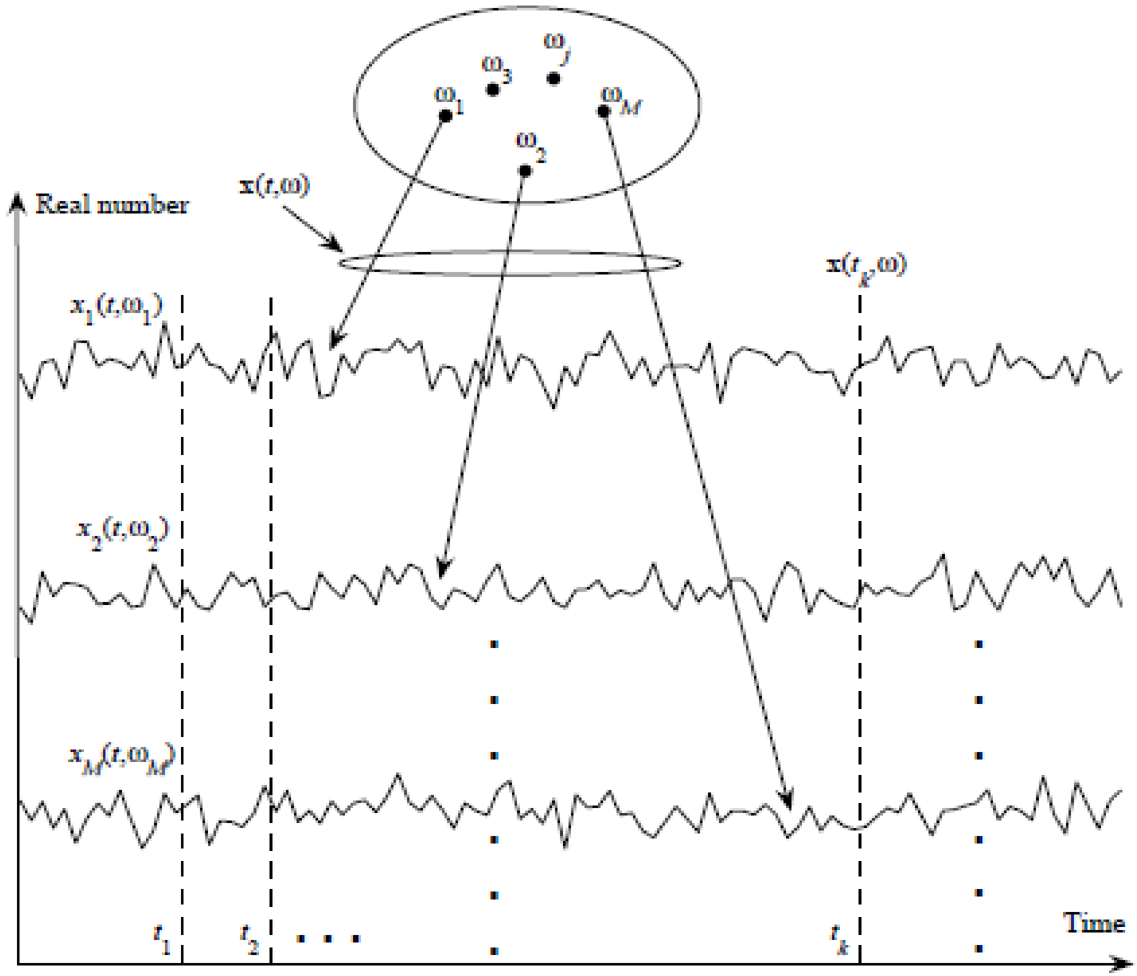
RANDOM PROCESS

- A random process is a collection of time functions, or signals, corresponding to various outcomes of a random experiment. For each outcome, there exists a deterministic function, which is called a sample function or a realization.



RANDOM PROCESS

- A mapping from a sample space to a set of time functions.



Random Process contd

- ***Ensemble***: The set of possible time functions that one sees.
- Denote this set by x(t), where the time functions x1(t, w1), x2(t, w2), x3(t, w3), . . . are specific members of the ensemble.
- At any time instant, t = tk, we have random variable x(tk).
- At any two time instants, say t1 and t2, we have two different random variables x(t1) and x(t2).
- Any realationship b/w any two random variables is called Joint PDF

Classification of Random Processes

- Based on whether its statistics change with time: the process is non-stationary or stationary.
- Different levels of stationary:
  - Strictly stationary: the joint pdf of any order is independent of a shift in time.
  - Nth-order stationary: the joint pdf does not depend on the time shift, but depends on time spacing